macQsimal	Title **Quantum-enhanced OPM benchmarking report**	Deliverable Number **D4.7**
Project Number **820393**		Version **Version 1.0**

H2020-FETFLAG-2018-2021

macQsimal

Miniature Atomic vapor-Cell Quantum devices for SensIng and Metrology AppLications

Deliverable D4.7

Quantum-enhanced OPM benchmarking report

WP4 – Miniature optically-pumped magnetometers

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Delivery date: 01.07.2022

Deliverable type: Report

Dissemination level: Public

This work has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 820393.

Revision History

Author Name, Partner short name	**Description**	**Date**
Charikleia Troullinou (ICFO)	Start of document, added sections “Light squeezing in atomic magnetometry”, Executive summary	13.12.2021
Morgan Mitchell (ICFO)	Added sections “Quantum noise and quantum enhancement”, “Spin squeezing in atomic magnetometry,” revised Executive summary	3.01.2022
Vito Giovanni Lucivero (ICFO)	Minor corrections	7.01.2022
Jakob Reichel (LKB)	Revision	17.06.2022
Charikleia Troullinou (ICFO), Morgan Mitchell (ICFO)	Final version	30.06.2022
Johannes Ripperger (accelCH)	Final formal checks and formatting	01.07.2022

Abbreviations

accelCH	Accelopment Schweiz AG *(macQsimal Beneficiary No. 3)*
AOC	Alignment-to-Orientation Conversion
ASN	Atomic Shot Noise
CNRS	Centre National de la Recherche Scientifique CNRS *(participating in macQsimal with the Laboratoire Kastler Brossel, LKB, macQsimal Beneficiary No. 6)*
FID	Free-Induction Decays
FWM	Four-Wave Mixing
GW	Gravitational Wave
HWP	Half-Wave Plate
ICFO	Fundacio Institut de Ciencies Fotoniques *(The Institute of Photonic Sciences, macQsimal Beneficiary No. 4)*
NMOR	Nonlinear Magneto-optical Rotation
OPM	Optically-Pumped Magnetometer
OPO	Optical Parametric Oscillator
PBS	Polarizing Beam-Splitter
PPKTP	Periodically Poled Potassium Titanyl Phosphate
PSN	Photon Shot Noise
PSR	Polarization Self-Rotation
SERF	Spin Exchange Relaxation-Free
SPDC	Spontaneous Parametric Down-Conversion
SQL	Standard Quantum Limit

Contents

1 Overview / Executive summary

This document reports the results of the quantum enhancement benchmarking exercise within WP4 of the macQsimal project. The benchmarking exercise took as objectives to 1) situate OPMs within the broader context of quantum enhancement efforts 2) identify performance metrics for quantum enhancement of optically-pumped magnetometers (OPMs) and 3) identify the state of the art and best practices as regards quantum enhancement applied to OPMs. To this end, the report first draws upon the example of quantum enhancement in optical interferometry to identify a plausible trajectory for the incorporation of quantum enhancement techniques into state-of-the-art practice with OPMs. The report then presents the main sources of quantum noise in OPMs, and the state of the art for techniques to reduce such noise. The report then specializes to the most advanced approach, the application of squeezed light, and in particular polarization squeezing, to enhance the performance of high-sensitivity OPMs. It makes a comparative survey of experiments that have employed squeezed light in different configurations of optical magnetometers, with various types of squeezed light generation. The report then describes the experimental progress in this direction within macQsimal, showing simultaneous enhancement of measurement bandwidth and high frequency sensitivity in an OPM with sub-pT sensitivity. A theoretical model that explains and predicts the twofold advantage is also described. Finally, the report describes related OPM methods that could be similarly enhanced.

2 Quantum noise and quantum enhancement

2.1 Standard quantum limit

Quantum noise, for example shot noise and noise from measurement back-action, is of practical concern for an increasing number of sensing and metrology technologies, including optical interferometers [1], atomic clocks [2] [3], optical magnetometers [4] [5,6] and a variety of spectroscopy techniques [7,8]. In general, the relative strength of quantum noise increases for smaller sensing systems, i.e., those employing fewer particles, and as technical and thermal noise are reduced. Consequently, a general trend toward miniaturized sensors, together with improvements in low-noise materials, electronics, lasers, and other OPM components, suggest that quantum noise will play an ever-larger role in determining the performance of sensing and metrology technologies.

Quantum noise is often described relative to the *standard quantum limit* (SQL), which can be broadly defined as the best performance achievable using non-entangled systems. In some applications such as optical interferometry, the SQL is known to set a lower limit on the quantum noise when using traditional light sources such as lasers. In these applications, lower noise levels can nonetheless be achieved by using squeezed light and other manifestations of photonic entanglement. *Quantum enhancement* is the application of squeezing and related methods to achieve performance beyond the SQL.

As an example, when measuring an optical phase ϕ with a linear interferometer, optical shot noise contributes a power spectral density $S_\phi(\nu) = 2\hbar\omega\langle \mathrm{P}\rangle^{-1}$, where $\hbar\omega$ is the photon energy and $\langle \mathrm{P}\rangle$ is the mean power of the beam. A similar shot noise limit applies to optical polarisation rotation, widely used in OPMs. Methods such as quadrature squeezing or polarisation squeezing can reduce the optical quantum noise below this level, with a consequent improvement in signal to noise ratio in sensing applications, e.g., gravitational wave (GW) detection or optical magnetometry.

We note that the SQL is in general implementation-dependent. In the above example, $S_\phi(\nu)$ contains $\hbar\omega$, the quantum of photon energy, and also $\langle \mathrm{P}\rangle$, the optical power used in the measurement. Consequently, one can reduce $S_\phi(\nu)$ at least three ways: by measuring with a longer wavelength, by measuring with higher average power, and by using optical squeezing. What combination of these options is most efficacious will depend on many technology-specific factors.

2.2 Trajectory of quantum enhancement in the optical domain

In GW detection, which is the best-known and best-developed application of quantum enhancement, the most efficacious strategy has evolved as both laser and squeezed light technology have advanced. Although squeezed light was proposed in 1981 as a method to improve GW detectors, and proof-of-principle demonstrations of optical squeezing were available by 1985, this technique was not used in real instruments until about 2010 [1]. During the intervening time, laser technology advanced, allowing greater powers to be used in these interferometers. Meanwhile, squeezed light technology advanced to make squeezed light available at wavelengths and sideband frequencies suitable for GW detectors [9,10]. Eventually, large gravitational-wave detectors reached optical powers that produced a number of deleterious optical nonlinearities, e.g., optomechanical instability [11–13], and it was no longer efficacious to increase average power. Roughly simultaneous with this development, quantum enhancement techniques became available to be integrated into these detectors [1] [2] [3].

This history of GW detection suggests a development trajectory in which “classical” improvements such as reduced technical noise and higher optical power are first deployed, while quantum enhancement techniques are developed and adapted to the conditions found in high-performance sensors. Quantum enhancement is then incorporated into practice when it becomes efficacious.

2.3 Quantum noise in atomic sensors

Most atomic sensors, and all OPMs, are composed of at least two interacting quantum systems: an atomic medium and the light that pumps and probes it. Quantum noise is contributed by both the atoms and the light. The optical shot noise contribution, leading to a white noise in a polarization rotation measurement, was described above.

Atomic quantum noise has distinct manifestations. *Spin projection noise* describes the intrinsic uncertainty of the atomic observables. For example, a collection of N spin-1/2 atoms can be described by collective spin operators that describe the net spin of the atoms. If the atoms are fully polarized along the x direction, and thus have a spin projection along x of $F_x = N/2$, this collective spin will have an uncertainty of $\delta F_y = \delta F_z = \frac{1}{2}\sqrt{N}$ for projections along the y and z directions. Spin projection noise, also mentioned as atomic shot noise (ASN), is associated with the structure of the quantum mechanical observables and introduces a random element into measurement of the spin.

In contrast, *quantum spin fluctuations* arise from stochastic processes that occur in the atomic medium, e.g., collisions or diffusion of atoms. These are not intrinsically connected to measurement of the spin observables and are not imposed by uncertainty relations. Quantum spin fluctuations are, nonetheless, a noise source of specifically quantum origin, because they are a consequence of the indivisible, atomic nature of the medium. Sometimes quantum spin fluctuations can be inferred from relaxation processes via the fluctuation-dissipation theorem.

In contrast to optical shot noise, which is white, quantum spin fluctuations typically contribute a noise spectrum that reflects the dynamical properties of the atoms. For example, in a magnetic field, an atomic spin ensemble will precess at the Larmor frequency, while also relaxing toward an unpolarized state due to a variety of relaxation mechanisms. When this spin system is driven by a time-varying input, e.g., by modulated optical pumping, the spin system responds resonantly, becoming more strongly polarized in response to a drive close to the Larmor frequency. Atomic collisions, and also the stochastic entry and exit of atoms from the ensemble, contribute a broadband drive, due to the short duration of collision events. This broadband drive, together with the resonant response of the spin system, lead to a spin noise concentrated at the response frequency of the medium.

In experimental contexts, the combined effects of spin projection noise and quantum spin fluctuations are referred to as *atomic shot noise* (ASN), and contribute to the Faraday rotation signal a noise power that scales as the density of atoms (whereas the signal power will scale as density of atoms squared).

2.4 Light-atom interactions and quantum noise

In OPMs and other quantum sensors, the atomic degrees of freedom respond to the environment and are indirectly measured by interaction with an optical beam. There is a great diversity of physical effects that arise at this interaction between light and atoms, with varying implications for quantum noise and quantum enhancement. For example, some OPMs detect atomic spin dynamics by resonant absorption of probe light [14,15], whereas other OPMs probe the atoms with off-resonance light, which experiences a dispersive effect such as polarization rotation [16]. Absorption of light efficiently destroys the photon-photon correlations that result in optical squeezing, however. As a consequence, an absorption-based OPM is an unlikely choice for quantum enhancement by this method. Absorption of light, and the attendant emission of light, are also stochastic process that can introduce noise into the spin degrees of freedom, so that also quantum enhancement methods based on spin squeezing are not naturally matched to absorption-based magnetometry.

Dispersive probing also has consequences for quantum noise. The first consequence is potentially very beneficial, as the non-disturbing measurement of a physical variable can result in a squeezed state, in which the post-measurement value is known with a precision beyond the SQL for that observable. Subsequent measurements can then reveal changes in that observable beyond the SQL for detection of such changes. Such non-disturbing measurements beyond the SQL are known as *quantum non-demolition* measurements [17,18], and are one of the main ways to produce spin squeezing and other sensitivity-improving quantum states in atomic systems. For example, in paramagnetic Faraday rotation, the polarization plane of the beam propagating along the z direction rotates by an angle proportional to the collective spin projection F_z, whereas F_z is not disturbed by the measurement. A Faraday rotation measurement with a precision below the uncertainty $\delta F_z = \frac{1}{2}\sqrt{N}$ (as described above), will leave the atomic spin system in a *spin squeezed state* with reduced uncertainty.

The same interaction that allows the light to be influenced by an atomic observable, also leads to *measurement back-action* effects on the atomic spins. In the example of Faraday rotation, the same interaction that produces rotation of the plane of polarization of the photons produces also a rotation of the spins about the F_z axis, by an angle proportional to the ellipticity of the probe light. In normal operation, linearly polarized probe light would be used for Faraday rotation probing, resulting in zero mean ellipticity and thus zero mean rotation of the spins. Nonetheless, even a perfectly polarized beam will have quantum fluctuations of the ellipticity, and these will produce a fluctuating rotation of the spins about F_z. A stronger measurement of F_z will inevitably result in greater input of random rotation by this means, a fact that is guaranteed if spin uncertainty relations are to be obeyed.

3 Spin squeezing in atomic magnetometry

Quantum projection noise is an important contributor to the performance of some state-of-the-art sensors, notably cold-atom and cold-ion atomic clocks [19–21]. In contrast, the highest-sensitivity OPMs, e.g., SERF magnetometers [22,23], are typically not limited by spin quantum noise. This is in part because these OPMs operate with high atomic densities and/or large vapor volumes, leading to a very large number of participating atoms and thus a very small SQL for the spin-precession angle. It is common for reports of OPM sensitivity to include a “projected sensitivity,” i.e., an estimate of the SQL for the magnetometer sensitivity, that is at least an order of magnitude below the true sensitivity. This indicates that other factors, e.g., magnetic noise of the OPM components or shielding, or technical noise of lasers or detectors, are currently limiting these sensitivities.

For other magnetometer types, in which the atom number is reduced, quantum projection noise can be significant, suggesting that quantum non-demolition measurement and spin squeezing could be beneficial [24–26]. Meanwhile, proof-of-principle experiments with small numbers of atoms have demonstrated magnetic sensitivity improvements through quantum non-demolition measurement and spin squeezing [25,27–30], and also more exotic techniques, e.g., cold collisions in Bose-Einstein condensates [31,32]. One experiment has even demonstrated spin squeezing of a SERF-regime vapour [33], suggesting that a SERF magnetometer with sufficiently low technical noise could be quantum enhanced by spin squeezing.

4 Light squeezing in atomic magnetometry

4.1 Review of the state of the art

A squeezed light enhanced magnetometer was first accomplished in 2010 [4] with polarization squeezing generated in Spontaneous Parametric Down-conversion (SPDC) in a sub-threshold optical parametric oscillator (OPO) cavity. The OPM operated by alignment-to-orientation conversion (AOC) in an ensemble of Rb atoms pumped only by the off-resonant probe light. The experiment was performed in an area of operating parameters for which the photon shot noise (PSN) was dominant, so that the suppression of photon shot noise due to squeezed light probing has been considerable. The magnetometer sensitivity was improved by 3.2 dB and reached nT/$\sqrt{\text{Hz}}$ sensitivity when polarization squeezed light was used for probing. The vapor cell of this experiment was at low temperature, a condition beneficial for preserving to a good degree the generated squeezing. At this low density any measurement backaction effect would have been negligible.

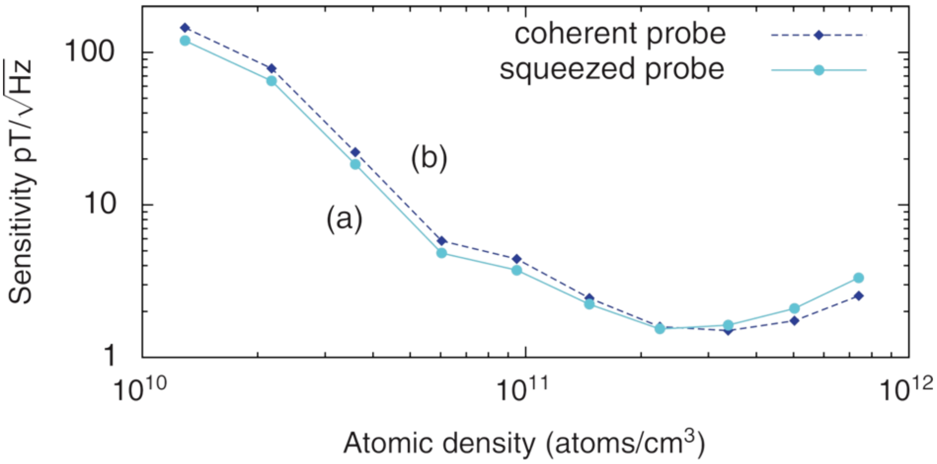

Figure 1: Sensitivity as a function of atomic vapor density in a single beam squeezed light enhanced magnetometer. Figure from Horrom et al. 2012 [5].

In 2012, a magnetometer with orders of magnitude improved sensitivity was realized by Horrom *et al.* [5]. Squeezed light was generated through polarization self-rotation in an atomic squeezer, a Rb cell placed before the magnetometer and along the propagation axis. This setup guaranteed the independent preparation of the input squeezed light. Even though the squeezer has been designed to have good performance at low frequencies, this has not been achieved due to laser technical noise. An even more important study has been performed in that experiment by scanning the vapor cell temperature and thus the atomic density. Operation at higher densities was experimentally proven to improve the sensitivity for the low density up to a critical density of about 2×10^{11} atoms/cm^3. As shown in Figure 1, squeezed light probing was beneficial only for atomic density up to that critical density. Moreover, the density-optimized sensitivity appears to be the same with or without squeezing, the only difference being the density at which this optimum occurs. The behaviour has been attributed to the backaction of the squeezed light on the atomic spin ensemble, which produces spin noise that becomes more important at high atomic density. At the time this work was published, it was not clear whether this backaction effect was a generic property of squeezed-light enhancement or was specific in some way to the squeezer and/or OPM strategy employed in this experiment.

In 2014, Otterstrom *et al.* [6] operated a single beam NMOR magnetometer, enhanced by squeezing simultaneously generated in the same vapor cell by means of four wave mixing. The magnetometry process took place at the same vapor cell including ^{85}Rb where a powerful pump gives rise to the non-linear process in ^{85}Rb and at the same time polarizes the alkali atoms. The detection of the sum and conjugate and probe signal could eliminate the classical noise below the standard quantum limit (SQL) by 4.7 dB. This experiment operated on a compact setup achieved sensitivity of 20 pT/ $\sqrt{\mathrm{Hz}}$. This compact configuration though did not allow decoupling and independent optimization of the squeezed light and the magnetometer.

A recent experiment in squeezed light magnetometry has been performed by Novikova *et al.* [34] to study the additional noise in polarization self-rotation (PSR). Through this process polarization squeezing can be generated at specific frequencies. This setup also included a single cell of non-polarized ^{87}Rb cell, and the effect of squeezed light gave ultimate noise reduction of 4 dB at 2×10^{11} atoms/cm^3. The sensitivity after the squeezing was 20 pT/ $\sqrt{\mathrm{Hz}}$ and it has been anticipated to improve as the density of Rb cell increases.

2021 has seen multiple experiments reported on this topic. Recent experiments [35] have shown improved squeezing (3.7dB) at low frequency range in a single beam low density atomic magnetometer. The reported sensitivity of about 20pT/ $\sqrt{\mathrm{Hz}}$ has been enhanced as probing with squeezed state of light generated also in a system based through an SPDC with good performance at low frequency ranges.

Light squeezing and NMOR magnetometry has been studied by Zhang *et al.* [36] in a single beam setup where squeezing may coexist with the magnetic sensing process depending on the probe power level. The study showed that the magnetic sensitivity has been optimized in the lower power regime where squeezing was not generated.

Table 1 summarizes the results of the experiments applying squeezed light to optically-pumped magnetometers.

Table 1: Experimental results on quantum enhancement of OPM sensitivity using squeezed light.

Reference	**Atomic magnetometer**	**Type of Squeezer**	**Sensitivity**	**Degree of squeezing**	**Operating frequency**
Wolfgramm *et al.* (2010)	NMOR	SPDC OPO	1 nT/ $\sqrt{\mathrm{Hz}}$	3.2 dB	120kHz
Horrom *et al.* (2012)	NMOR	PSR	2 pT/ $\sqrt{\mathrm{Hz}}$	2 dB	200 kHz
Otterstrom *et al.* (2014)	NMOR Single beam	FWM	19.3 pT/ $\sqrt{\mathrm{Hz}}$	4.7 dB	700 kHz
Novikova *et al.* (2015)	Nonlinear Faraday effect	PSR	3 pT/ $\sqrt{\mathrm{Hz}}$	2 dB	170 kHz
Bai *et al.* (2021)	NMOR	PSR	20 pT/ $\sqrt{\mathrm{Hz}}$	3.7 dB	10 kHz
Troullinou *et al.* (2021)	Bell Bloom OPM, Faraday rotation	SPDC OPO cavity	300 fT / $\sqrt{\mathrm{Hz}}$	2.3 dB	30kHz

4.2 Sub pT/sqrt(Hz) squeezed-light enhanced magnetometer

The macQsimal activity on quantum enhancement of OPM performance using optical squeezing has shown that these quantum enhancement techniques are beneficial, even in the high-density conditions encountered with high-sensitivity OPMs. At these high densities, both optical shot noise and spin quantum noise are important in determining the OPM equivalent magnetic noise. Also, the interaction of the probe with the spin system, including measurement back-action, cannot be neglected. It is thus a non-trivial question whether probe squeezing can enhance sensitivity or other characteristics of the OPM in these conditions, as already made evident by prior work [5].

In this work [5], squeezed vacuum is generated in an OPO cavity through SPDC. This is mode matched with the strong LO field with perpendicular polarization to generate polarization squeezed light of about 2.3 dB below the standard quantum limit (SQL).

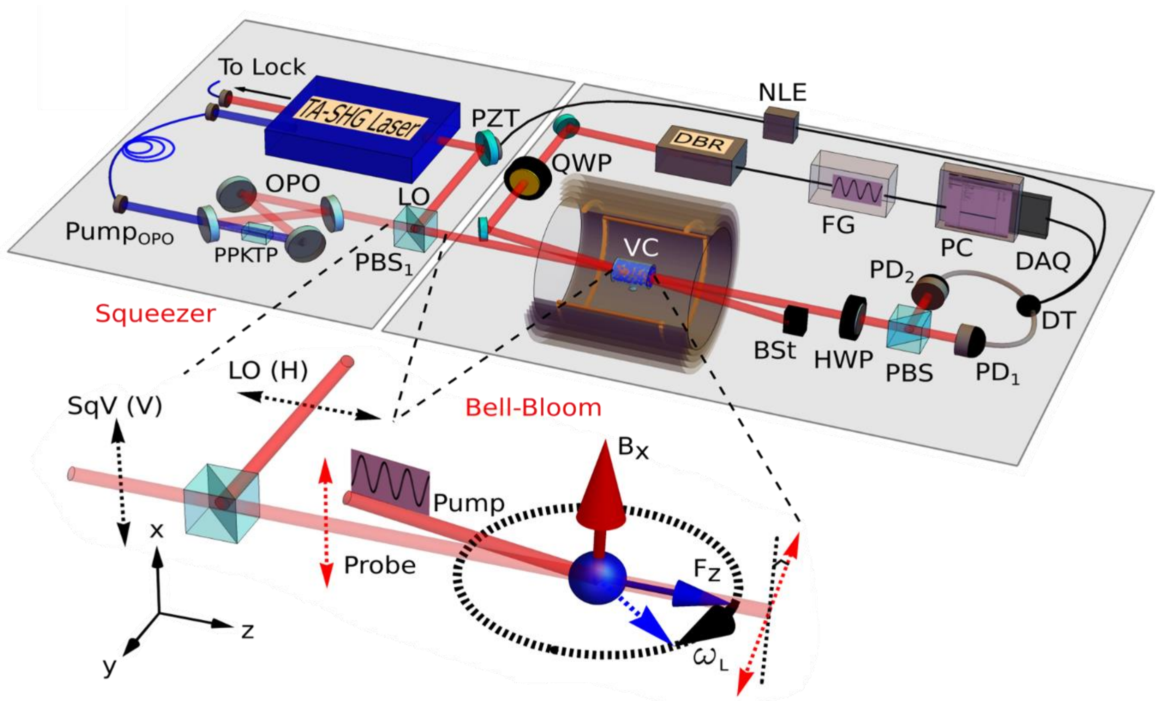

Figure 2 ***Squeezed-light Bell-Bloom OPM [38]****.* ***Experimental setup****. TA-SHG, Tapered Amplified Second Harmonic Generator; OPO, Optical Parametric Oscillator; PPKTP, Nonlinear crystal; LO, Local Oscillator; PBS, Polarizing Beam Splitter; QWP - Quarter Wave-plate; VC - Vapor Cell; BSt - Beam stopper; HWP - Half Wave-Plate; PD - Photodiode; DTIA - Differential Transimpedance Amplifier; DAQ - Data Acquisition; FG - Function Generator; NLE - Noise Lock Electronics,* ***"Bell-Bloom" Inset:*** *Due to the magnetic field Bx atomic spins precess at the Larmor frequency ω_L in the transverse plane. Synchronously modulated optical pumping maintains the atomic spin polarization. A linearly polarized cw probe undergoes paramagnetic Faraday rotation.* ***"Squeezer" Inset****: Vertically-polarized squeezed vacuum is combined with horizontally-polarized LO on a polarizing beam splitter to generate a polarization squeezed probe.*

The setup depicted in Figure 2 employs Bell Bloom excitation [37] in a bias field of about 4.3 µT. In this magnetic field, an order of magnitude smaller than Earth's magnetic field, the quantum noise is dominant across the whole spectrum, with ASN being dominant below about 200 Hz, PSN being dominant above that frequency. This is evident in Figure 3. When probing with polarization squeezed light the photon shot noise drops by about 2 dB. This noise reduction is evident and advantageous and in the PSN limited regime as well as in the transition region. As described in detail in the theoretical model of this work [38], the ASN and the magnetic responsivity are functions of frequency showing a low pass filter trend with cutoff frequency determined by the coherence time T_2. In contrast, the PSN is constant through the spectrum. When applying squeezed light, PSN drops and the crossover frequency between it and ASN increases. This results in increase of 17% in the measurement bandwidth, defined as the frequency at which the sensitivity deteriorates by 3dB from its optimum. The sensitivity spectrum is given by the equation

$$\boldsymbol{S_B(\omega)} = \left(\frac{\boldsymbol{dv}}{\boldsymbol{dB}}\right)^{-2} \frac{\boldsymbol{S_v(\omega)}}{\left|\boldsymbol{\hat{R}(\omega)}\right|^2} \qquad \textit{Eq. 1}$$

Where $\frac{dv}{dB}$ is the slope of the quadrature component v over the magnetic field, $S_v(\omega)$ the noise spectrum and $|\hat{R}(\omega)|$ the normalized magnetic responsivity. As the polarization noise spectrum changes as shown in Figure 3b when probed with squeezed light, the noise in the high frequency regime clearly drops, implying an improved sensitivity. For example, the sensitivity improves by 16% at an analysis frequency of 500 Hz, see inset of Figure 4.

Furthermore, the low frequency regime does not show any excess in noise, although the OPM is operated in a high-density regime of about 10^{13} atoms/cm^3. This is an experimental verification that in such magnetometer scheme, where the probe beam is perpendicular to the main applied field, measurement backaction is naturally evaded. See supplemental material of reference [38].

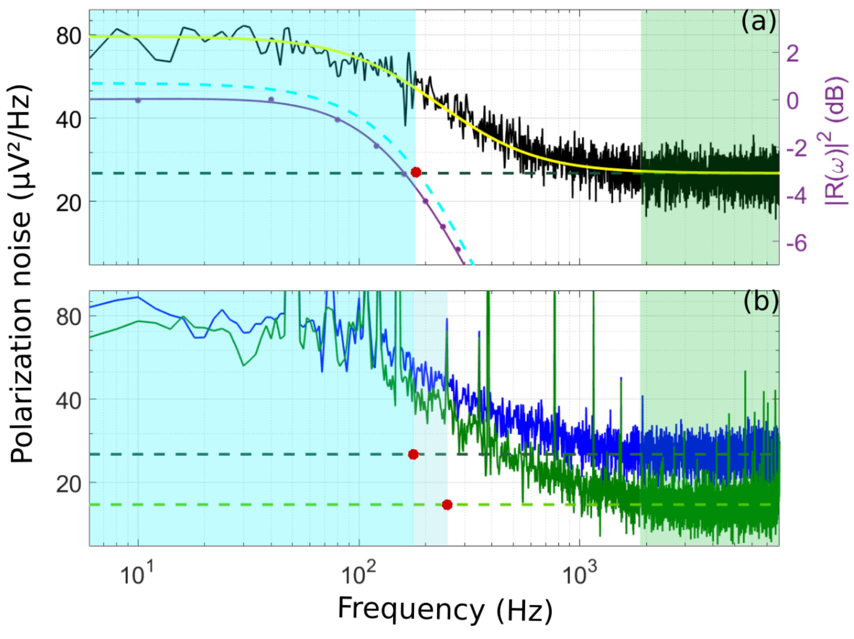

*Figure 3: **Polarization rotation noise after demodulation**. Upper graph: **Spin noise for unpolarized atoms.** Solid black curve: experimental data. The quantum noise levels estimated from the non-polarized spin noise spectrum (dashed green— PSN, dashed cyan —ASN) define the spin projection noise (cyan) and photon shot noise (green) limited areas and the intermediate transition region (white). The purple dots and purple curve indicate the measured normalized magnetic response at different frequencies and the fit for the magnetic responsivity. Lower graph: **Magnetometer noise for polarized atoms.** Quantum noise is dominant. At high frequencies, the noise level is reduced by 1.9 dB for squeezed light (green), with respect to the coherent (blue) probing. The dashed lines and the red dots depict estimates of photon shot noise level and cross-over frequencies when the squeezer is on and off, respectively. Taken from [38].*

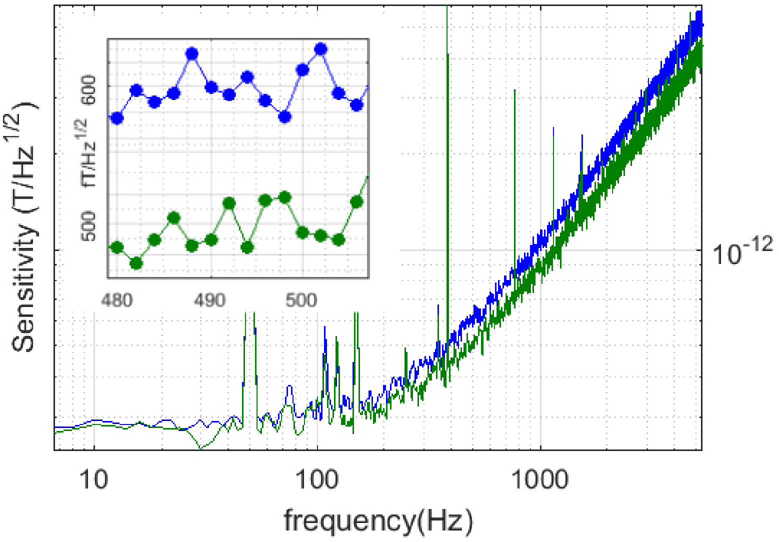

*Figure 4: **Magnetic sensitivity spectra** for the Bell Bloom magnetometer probed with coherent (blue) and squeezed light (green). Taken from [38]).*

4.3 Magnetometer schemes amenable for squeezed light application

A number of OPM strategies, including Bell-Bloom and FID, subject the spins to repeated non-destructive measurement as the spins precess. The first requirements for polarization squeezed light to be successfully applied and enhance the atomic sensor's performance for at least some part of the spectrum is that it operates under photon shot noise limited conditions. If, in addition, the measurement strategy evades measurement back-action, as in the Bell-Bloom and FID strategies, the beneficial impact will be evident through the spectrum.

A few OPM strategies are already at the point that squeezed light could be directly employed. As an example, we can consider atomic comagnetometers devoted to search for new physics [39] [40]. From more practical applications we could distinguish the gradiometer configuration of [41]. The sensor is based on FID protocol, transiently pumped with an exceptionally strong pump beam. It is probed with an off-resonance beam and it shows nearly quantum noise limited performance while operating in magnetic fields from 5µT to 50µT, Earth's magnetic field magnitude. The application of squeezed light is compatible with this sensor and it could exceed the described sensitivity of 14 fT/ $\sqrt{\mathrm{Hz}}$, a record for sensitivity in this demanding area of parameters.

Concerning the additional measurement bandwidth enhancement, it is expected to show in all those sensors in which high atomic densities are employed. In other words, in all sensors for which ASN surpasses PSN in some part of the spectrum.

Squeezed-light probing is additionally compatible with and complementary to other methods to enhance sensitivity and measurement bandwidth. Some of them include spin exchange relaxation suppression, pulsed protocols [42], multi pass geometries [43] and closed-loop techniques [44]. There is also possibility to recover signal components beyond the natural bandwidth of a sensor using Kalman filtering techniques [45]. These methods are limited by the optical shot noise and would be enhanced as squeezing reduces this noise source.

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